

# IDS 101 Doing Mathematics

## Today

1. Share your theorem and proof with someone nearby. Read each others' proofs carefully and critically to see if you find the argument understandable and convincing.
2. Groups! Start meeting with me next week. Schedule some times.
3. Some questions about cc-coloring (see below for reminders):
  - (a) Can you determine  $\chi_c(G)$  for the following graph families?
    - i. Paths
    - ii. Cycles
    - iii. Trees
    - iv. Complete graphs
    - v. Bipartite graphs/complete bipartite graphs
  - (b) Is there a graph  $G$  such that  $\chi(G) < \chi_c(G)$ ?
  - (c) Is there a graph for which choosing a single color for a clique takes more colors than choosing all different colors for that clique?
  - (d) What if we change the rules so that only cliques with at least 4 (or 5, or 6, or ...) vertices can be colored the same?
  - (e) If you add a new edge to a graph, the chromatic number can never go down. Argue that this is true.
  - (f) Is it possible to get a smaller cc-chromatic number for a graph by adding a new edge?
  - (g) What other questions do you come up with that we could try to answer about cc-coloring?
  - (h) **Definition:** Call a coloring of a graph  $G$  a **clique-choice** coloring (cc-coloring) if it obeys the following rules:
    - i. If vertices belong to a clique of size 3 or greater, they are either all different colors or all the same color; and
    - ii. otherwise, adjacent vertices are different colors as in a proper coloring.The **clique-choice chromatic number** of a graph  $G$  is the smallest number of colors for a which a valid cc-coloring exists, denoted  $\chi_c(G)$ .
  - (i) Monday: we will watch "The Proof" and discuss it on Wednesday.